10.3.7 Integrování substituční metodou II

Předpoklady: 10306

Př. 1: Vypočti:

a) $\int \frac{3}{\sqrt{5x-1}}dx$ b) $\int x^2\sqrt[3]{1-x^3}dx$ c) $\int \frac{1}{2-5x}dx$

a)

$$\int \frac{3}{\sqrt{5x-1}}dx = \int \frac{1}{5}\cdot\frac{3}{\sqrt{5x-1}}5dx = \frac{3}{5}\int \frac{1}{\sqrt{t}}dt = \frac{3}{5}\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+C = \frac{6}{5}\sqrt{t}+C = \frac{6}{5}\sqrt{5x-1}+C$$

$t = 5x-1 \Rightarrow dt = 5\,dx$

b)

$$\int x^2\sqrt[3]{1-x^3}dx = \int\left(-\frac{1}{3}\right)\left(-3x^2\right)\sqrt[3]{1-x^3}dx = -\frac{1}{3}\int\sqrt[3]{t}\,dt = -\frac{1}{3}\frac{t^{\frac{4}{3}}}{\frac{4}{3}}+C = -\frac{1}{4}\sqrt[3]{t^4}+C$$

$t = 1-x^3 \Rightarrow dt = -3x^2\,dx$

$$= -\frac{1}{4}\sqrt[3]{\left(1-x^3\right)^4}+C$$

c)

$$\int \frac{1}{2-5x}dx = \int\left(-\frac{1}{5}\right)\frac{1}{2-5x}(-5)\,dx = -\frac{1}{5}\int\frac{1}{t}dt = -\frac{1}{5}\ln|t|+C = -\frac{1}{5}\ln|2-5x|+C$$

$= t = 2-5x \Rightarrow dt = -5\,dx$

Př. 2: Vypočti:

a) $\int \cos^3 x\cdot\sin x\,dx$ b) $\int \frac{\ln x}{2x}dx$ c) $\int \frac{3}{(3-2x)^5}dx$

¨ a) $\int \cos^3 x\cdot\sin x\,dx = -\int \cos^3 x\cdot(-\sin x)\,dx = -\int t^3 dt = -\frac{t^4}{4}+C = -\frac{\cos^4 x}{4}+C$

$t = \cos x \Rightarrow dt = -\sin x\,dx$

b) $\int \frac{\ln x}{2x}dx = \frac{1}{2}\int \ln x\frac{1}{x}dx = \frac{1}{2}\int t\,dt = \frac{1}{2}\frac{t^2}{2}+C = \frac{\ln^2 x}{4}+C$

$t = \ln x \Rightarrow dt = \frac{1}{x}dx$

c) $\int \frac{3}{(3-2x)^5}dx = -\frac{3}{2}\int\frac{1}{(3-2x)^5}(-2)\,dx = -\frac{3}{2}\int\frac{1}{t^5}dt = -\frac{3}{2}\frac{t^{-4}}{-4} = \frac{3}{8(3-2x)^4}$

$t = 3-2x \Rightarrow dt = -2\,dx$

Př. 3: Vypočti:

a) $\int \sin(2x-\pi)\,dx$ b) $\int ae^{-x}\,dx$ c) $\int \cos\left(\frac{2}{3}x+\frac{\pi}{2}\right)dx$

a) $\int \sin(2x-\pi)\,dx = \frac{1}{2}\int \sin(2x-\pi)\,2dx = \frac{1}{2}\int \sin t\,dt = \frac{1}{2}(-\cos t)+C = -\frac{1}{2}\cos(2x-\pi)+C$

$t = 2x-\pi \Rightarrow dt = 2\,dx$

b) $\int ae^{-x}\,dx = -a\int e^{-x}(-1)\,dx = -a\int e^{t}\,dt = -a\cdot e^{t}+C = -a\cdot e^{-x}+C$

$t=-x \Rightarrow dt = (-1)\,dx$

c) $\int \cos\left(\frac{2}{3}x+\frac{\pi}{2}\right)dx = \frac{3}{2}\int \cos\left(\frac{2}{3}x+\frac{\pi}{2}\right)\frac{2}{3}dx = \frac{3}{2}\int \cos t\,dt = \frac{3}{2}\sin t + C = \frac{3}{2}\sin\left(\frac{2}{3}x+\frac{\pi}{2}\right)+C$

$t = \frac{2}{3}x+\frac{\pi}{2} \Rightarrow dt = \frac{2}{3}dx$

Př. 4: Petáková:
strana 164/cvičení 89 b) d) f)

Shrnutí: Substituční metodu můžeme při integraci použít pokud se nám podaří vytvořit v integrálu derivaci substituované funkce.